647510 **MSC3MatC302x** **Seat No :________**

M.Sc.(Maths.) Semester - 3 (*CBCS*) Examination
Oct/Nov-2022 (NEW COURSE)
Functional Analysis (Core (New))

Time: 2:30 Hours **Marks: 70**

Instructions:

1. All questions are compulsory.
2. Figures to the right indicate marks.

Q.1(a) **Answer the following.** **[4]**

(1) Define norm and give any one norm on $\mathbb{R}^2$.

Q.1(b) **Answer any two questions.** **[10]**

(1) Prove that every finite dimensional normed space is complete.

(2) Show that $\mathbb{R}^n$ is a normed linear space.

(3) Show that a normed space $(l^p, \| \ \|_p)$ is a Banach space, where $1 \leq p < \infty$.

Q.2(a) **Answer the following.** **[4]**

(1) Define Cauchy and convergent sequence in a metric space and give relation between it.

Q.2(b) **Answer any two questions.** **[10]**

(1) Let N be a normed linear space. Prove that $\left| ||x|| - ||y|| \right| \leq ||x - y||$, for $x, y \in N$.

(2) Show that any two norms on a finite dimensional normed space are equivalent.

(3) Prove that if $(X, \| \cdot \|)$ is finite dimensional normed space and M is subset of X then M is compact iff M is closed and bounded subset of X.

Q.3(a) **Answer each of the followings.** **[4]**

(1) Define Banach space.

(2) State the required condition on N and N' for $\mathcal{B}(N, N')$ to be a Banach space.

Q.3(b) **Answer any two questions.** **[10]**

(1) Show that the conjugate space of L^p $(1 < p < \infty)$ is L^q.

(2) Prove that subspace Y of Banach space X is complete if Y is closed in X.

(3) State and prove closed graph theorem.

Q.4(a) Define reflexive spaces and show that l^2 is reflexive. **[4]**

Q.4(b) **Answer any two questions.** **[10]**

(1) State and prove parallelogram law.

(2) If x and y are two vectors in a Hilbert space, then prove that $|\langle x, y \rangle| \leq \|x\| \|y\|$.

(3) Prove that every Hilbert space is reflexive.

Q.5(a) State Holder's and Minkowski's inequality for sums. **[4]**

Q.5(b) **Answer any two questions.** **[10]**

(1) Explain natural imbedding of a normed linear space N into N^{**}.

(2) A bounded linear operator $P: H \to H$ on a Hilbert space H is a projection if and only if P is self-adjoint and idempotent.

(3) What is $H^\perp$ and $\{0\}^\perp$ for a Hilbert space H? Explain in detail.

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